Shrikrishna Shikshan Sansth's

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DEPARTMENT OF MATHEMATICS .

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Introduction.

First of all let me introduce myself My name is Tejaswini Anand Dangare and the institution name where I am currently study in the name is Shri Krishna Mahavidayalaya, Gunjoti.

I Study in M.Sc-II (Maths) this report is about the subject wise project work. Subject name is Numerical & Linear Integral Equation and my alloted topic Name was

"studies on Eigenvalues and Eigenfunction."

I Completed my project work as shown in this report.

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* Basic Deffinitions:-

* Linear Integral Equation.

The equation is a linear integral equation because ϕ appears in a linear form (i.e., we do not have terms like ϕ^2)

If $a=0$ then we have a Fredholm integral equation of the first kind In these equations the unknown appears only in the integral term.

Integral equation types:-

1) Volterra equation of the first kind.
2) Volterra equation of the second kind.
3) Fredholm equation of the first kind.
4) Fredholm equation of the second kind.

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* Studies on Eigenvalues and Eigenfunction.

* Eigenvalues :-

Eigenvalues are the special set of scalar values that is associated with the set of linear equations most probably in the matrix equations. The eigenvectors are also termed as characteristic roots.

It is a non-zero vector that can be changed at most by its scalar factor after the application of linear transformations.

$$Ax = \lambda x$$

The number or scalar value "λ" is an eigenvalue of A.

For every real matrix, there is an eigenvalue. Sometimes it might be complex. The existence of the eigenvalue for the complex matrices is equal to the fundamental theorem of algebra.

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* Eigenfunction :-

In mathematics an eigenfunction of a linear operator D defined on some function space is any non-zero function f in that space that when acted upon by D, is only multiplied by some scaling factor called an eigenvalue. As an equation this condition can be written as

$$Df = \lambda f$$

for some scalar eigenvalue λ. The solutions to this equation may also be subject to boundary conditions that limit the allowable eigenvalues that limit the allowable eigenvalues and eigenfunctions.

An eigenfunction is a type of eigenvector.

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* Eigenvalues and Eigenfunctions :-

If we write the homogeneous Fredholm equation as.

$$\int_a^b K(s,t)\, g(t)\, dt = \mu g(s) \quad \cdots \quad \mu = \frac{1}{\lambda}.$$

we have the classical eigenvalue or characteristic value problem. μ is the eigenvalue and $g(t)$ is the corresponding eigenfunction or characteristic function since the linear integral equations are studied in the form (1.1.10).

it is λ and not $1/\lambda$ which is called the eigenvalue.

* Studies on Eigenvalues and Eigenfunction.

* Examples :-

* Find the Eigenvalues ~~for~~ and eigenfunction for the differential equation

$$y'' + \lambda y = 0$$

with the following boudary conditions

a) $y'(0) = 0 \quad , \quad y'(1) = 0$

→ there are no non-trivial solutions unless $\lambda > 0$.

We write $\lambda = K^2$ (with $K \neq 0$) & (*) becomes

$y'' + K^2 y = 0$, with solution

$y = A \cos(Kx) + B \sin(Kx)$

$y' = -AK \sin(Kx) + BK \cos(Kx)$

$y'(0) = 0$ gives $B = 0$ & hence.

$y = A \cos(Kx)$

using

$y(1) = 0$ gives

$A \cos K = 0$ for non-trivial

$\cos K = 0 \implies K = \frac{(2n-1)\pi}{2} \quad n = 1, 2, 3.$

The corresponding eigenvalues & eigenfunctions are

$$\lambda_n = \frac{(2n-1)^2 \pi^2}{4}, \quad y_n = \sin\left[\frac{(2n-1)\pi x}{2}\right], \quad n = 1, 2, 3 \cdots$$

* Vote of Thanks:-

In M.Sc - II (Maths) for Sem - IV (2021 - 2022) Student can do the all 05 Subject sectional work. Linear Integral Equation Subject Numerical Analysis my allotted project topic name was "Studies on Eigenvalues and Eigenfunction."

I would like to thanks our respected principal and always supportive professor Staff to Complete the project they should always supportive and motivat me.

So, Thank you all !

* References :-

1) Wikipedia

2) google.

3) Linear Integral Equation Notes

4) Linear Integral Equation - by Ram P Kanwal