Date 22/06/2022 Page:

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Topic :

Page : ....

Date : ....

INDEX

Sign: ....

Introduction

In mathematics, an eigenfunction of a linear operator D defined on some function space that, when acted upon by D, is only multiplied by some scaling factor called an eigenvalue. As an equation, this condition can be written as

$$Df = \lambda f$$

for some scalar eigenvalue λ. The solutions to this equation may also be subject to boundary conditions that limit the allowable eigenvalues and eigenfunctions.

Date / /20 Page:

The world of Eigenvalues-eigenfunctions:

An operator A operates on a functions which when operated by the operator produces the same function modified only multiplied by a constant factor.

For every such a function is called the eigenfunction of the operator and the constant modifier is called its corresponding eigenvalue. A eigenvalue is just a number. Real or complex

A typical eigenvalue equation would look like

$$Ax = \lambda x$$

~~ope~~ Here, the matrix or the operator A operates on a vector x producing an amplified or reduced over λx. Here the eigenvalue λ belongs to eigenfunction

Sign:

Eigenfunctions:

In general, an eigenvector of a linear operator D defined on some vector space is a nonzero vector in the domain of D that, when D acts upon it is simply scaled by some scalar value called an eigenvalue. In the special case where D is defined on a function space the eigenvectors are refered to as eigenfunctions. That is a function f is an eigenfunction of D if it satisfies ~~the~~ equation

$$Df = \lambda f,$$

Where λ is scalar.

The solutions to Equation may also be subject to boundry conditions.

Date / /20 Page:

Each value of λ corresponds to one or more eigenfunctions. If multiple linearly independent eigenfunctions have the same eigenvalue, the eigenvalue is said to be degenerate and the maximum number of linearly independent eigenfunctions associated with the same eigenvalue is the eigenvalue's degree of degeneracy.

Link to eigenvalues and eigenvectors of matrices.

Eigenfunctions can be expressed as column vectors and linear operators can be expressed as matrices, although they may have infinite dimensions. As a result, many of the concepts related to eigenvectors of matrices carry over to

Sign:

the study of eigenfunction.

Define the inner product in the function space on which D is defined as

$$\langle f, g \rangle = \int_{\Omega} f^{*}(t)\, g(t)\, dt,$$

integrated over some range of interest for t called Ω. The $*$ denotes the complex conjugate.

Suppose the function space has an orthonormal basis given by the set of functions $\{u_1(t), u_2(t), \dots, u_n(t)\}$ where n may be infinite. For the orthonormal basis,

$$\langle u_i, u_j \rangle = \int_{\Omega} u_i^{*}(t)\, u_j(t)\, dt = \delta_{ij} = \begin{cases} 1 & i= \\ 0 & i\neq \end{cases}$$

where δ_{ij} is the Kronekker delta and can be thought of as the elements of the identity matrix

Eigenvalues and eigenfunctions of Hermitian operators:

Many of the operators encountered in physics are Hermitian. Suppose the linear operators D acts on a function space that is a Hilbert space with an orthonormal basis given by the set of functions $\{u_1(t), u_2(t), \dots u_n(t)\}$ where n may be infinite. In this basis, the operator D has a matrix representation A with elements

$$A_{ij} = \langle u_i, Du_j \rangle = \int_{\Omega} dt\, u_i^*(t) D u_j(t)$$

integrated over some range of interest for t denoted Ω.

By anology with Hermitian matrices, D is a Hermitian operator if $A_{ij} = A_{ji}$

Date / /20 Page:

Its eigenvalues are real
Its eigenfunctions obey an orthogonality condition.

The second condition always holds for $\lambda_i \neq \lambda_j$. For degenerate eigenfunctions with the same eigenvalue λ_i, orthogonal eigenfunctions can always be chosen that span the eigenspace associated with λ_i, for example by using the Gram schmidt process.

For many ~~Hermitian~~ operators, notably Sturm Liouville operators, a third property is
As a consequence, in many important cases, the eigenfunctions of the Hermitian operator form an orthonormal basis.

Sign:

Date / /20 Page:

Applying the laws of mechanics to infinitesimal portions of the string, the function h satisfies the partial differential equation

$$\frac{\partial^2 h}{\partial^2 t^2} = c^2 \frac{\partial^2 h}{\partial x^2}$$

which is called the wave equation. Here c is a constant speed that depends on the tension and mass of the string.

If we assume that $h(x,t)$ can be written as the product of the form $X(x)T(t)$. we can ~~pform~~ a pair of ordinary differential equations:

$$\frac{d^2}{dx^2} X = -\frac{\omega^2}{c^2} X, \quad \frac{d^2 T}{dt^2} = -\omega^2 T$$

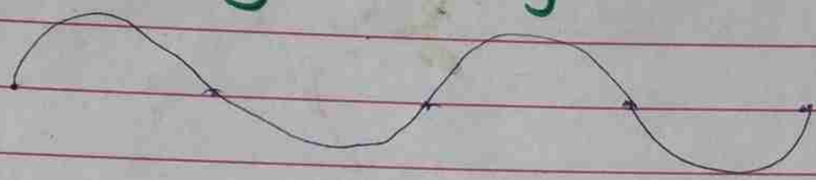
Each of these is an eigenvalue equation with eigenvalues $-\frac{\omega^2}{c^2}$ and $-\omega^2$, respect. For any values of ω and c, the equations are satisfied by the functions.

$$X(x) = \sin\left(\frac{\omega x}{c} + \varphi\right), \quad T(t) = \sin(\omega t)$$

Sign:

Applications

Vibrating strings:



The shape of a standing wave in a string fixed at its boundaries is an example of an eigenfunction of a differential operator. The admissible eigenvalues are governed by the length of the string and determine the frequency of oscillation.

Let $h(x,t)$ denote the transverse displacement of a stressed elastic chord, such as the vibrating strings of a string instrument, as a function of the position x along the string and of time t.

Date / /20 Page:

Schrodinger equation:

In quantum mechanics the Schrodinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(r,t) = H\psi(r,t)$$

with the Hamitonian operator

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(r,t)$$

can be solved by seperation of variables if the Hamitonian does not depend explicitly on time. In that case, the wave function $\psi(r,t) = \phi(r)T(t)$ leads to the two differential equations.

$$H\varphi(r) = E\varphi(r)$$

$$i\hbar \frac{\partial T(t)}{\partial t} = E\,T(t)$$

Both of these differential equations are eigenvalue equations with eigenvalue E.

Sign:

Date / /20 Page:

Signals and systems.

In the study of signals and systems, an eigenfunction of a system is a signal f(t) that when input into the system, produces a response $y(t) = \lambda f(t)$, where λ is a complex scal Eigenfunction.

Properties of Eigenvalues

- For a scalar matrix:

If A is a square matrix and λ is an eigenvalue of A. Then $a\lambda$ is an eigenvalue of A.

- For matrix power-

If A is square matrix and λ is an eigenvalue of A and $n \geq 0$, is an integer, then

Sign:

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PPFG 01

Date / /20 Page

Result

When an operator operating on a function results in a constant times the function the function is called an eigenfunction of the operator and the constant is called the eigenvalue.