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Studies on Canonical Forms

★ Introduction:

The development of the phase-variable canonical form for single-input linear controllable systems has been an active area of research to be an extremely convenient starting point for certain control design problems and partly it is because canonical forms are mathematically intriguing in their own right.

Unlike the single-variable case, the corresponding canonical forms for multivariable systems are not unique. This lack of uniqueness not only tends to make their derivative more difficult but also forces the design engineer, faced with a practical application, to determine the best from the several possibilities.

★ Canonical Forms:

A change of co-ordinates from state vector x to y defined by $y = Sx$ transforms the system from

$$\dot{x} = Ax + Bu \qquad (1)$$

to

$$\dot{y} = SAS^{-1}y + SBu$$

Appropriate choices of S lead to canonical forms of the system. Two basic canonical forms are developed from the matrix P constructed. There are variations possible within each of the two basic forms since there are variations possible in the choice of P. Each choice of P, however, leads to two basically different canonical forms.

The first canonical forms is produced by setting $S = P^{-1}$, Simple matrix verifies that the system is then transformed to the form.

with $$\dot{y} = \tilde{A}y + \tilde{B}u$$

$$\tilde{A} = \begin{bmatrix} 0 & 0 & \cdots & x & x & x \\ 1 & 0 & \cdots & x & \vdots & \vdots \\ 0 & 1 & \cdots & x & & \\ \vdots & & & & & \\ & & & & x & \\ & & & & x & \\ & & & & \vdots & \\ & & & & 0 \cdots & x \\ & & & & 1 \cdots & x \\ & & & & 0 & 1x \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & & \vdots \\ & 1 & & \\ & 0 & & \\ & \vdots & & 1 \\ & & & 0 \\ 0 & & & \vdots \\ & & & 0 \end{bmatrix}$$

The system may be considered as composed of fundamental companion matrices located in blocks along the diagonal. The x's in the matrix represent possible non-zero elements and, except for the indicated 1s, these occurs only in the columns corresponding to the right-hand edge of a fundamental companion matrix.

Different choices of P lead to different size and number of companion matrices as well as different values for the non-zero elements. If P were chosen according to the first plane, the x's in a given column of $\tilde{A}$ would be zero below the companion matrix corresponding to the represent a subsystem coupled to other sub-systems.

The second type of canonical form is more useful than the first but is somewhat more difficult to derive.

* Special Canonical form:

If the P matrix is constructed according to the second special selection plan of the $\tilde{A}$ matrix will appear as follows but $\tilde{B}$ reduces to

$$\tilde{B} = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ 0 & & & & \\ \vdots & & & & \\ 1 & x & \cdots & & x \\ \vdots & & & & \\ 0 & 0 & \cdots & & 0 \\ \vdots & & & & \\ \vdots & 1 & x & \cdots & x \\ \vdots & & & & \vdots \\ \vdots & & & & 0 \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

which is most easily verified by (tedious) inspection rather than by algebraic manipulation.

It follows that

$$\tilde{B} = \hat{B} C$$

where

$$\hat{B} = \begin{bmatrix} 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ \vdots & \vdots & \\ 1 & 0 & \cdots \\ 0 & & \vdots \\ 0 & & \\ \vdots & 1 & \\ \vdots & 0 & 0 \\ & \vdots & \vdots \\ 0 & \vdots & \end{bmatrix}$$

and C is an $r \times r$ upper-triangular matrix with is on the main diagonal.

Since C is invertible, a new but equivalent set of system inputs can be defined as

$$v = Cu$$

with respect to these inputs the system equation take the particular nice form.

$$\dot{y} = \tilde{A}y + \hat{B}v.$$

- Theorem 1:

Suppose the system is controllable with controllability index v_c. Then there is a non-singular transformation of the state vector & a non-singular transformation of the input vector which reduce the system to a coupled set of r single-input subsystems. Each subsystem is of order v_c or less & is in the standard single-input phase-variable form. Furthermore, all additional coupling enters a sub-system at its input.

- Lemma 1:

Unless a vector have already been selected and retained in the above, there is a +ve vector of the form $A^i b_k$ where all lower powers of A times b_k have been retained, which is linearly independent of all previously selected vectors.

Proof:

Suppose that the selected vectors are:

$b_1, Ab_1, \cdots, A^{q_1} b_1, b_2, Ab_2 \cdots, A^{q_2} b_2, b_3 \cdots A^{q_r} b_r$

and that each of the vectors

$A^{q_{1+1}} b_1, A^{q_{2+1}} b_2, \cdots, A^{q_{r+1}} b_r$

are linearly independent on the selected vectors so that the process terminates. It then follows by the induction argument that all other vectors in the controllability matrix are linearly dependent on the selected vectors. This in turn implies that either the rank of the controllability matrix is less than n or there are n independent vectors in the plane.

The vector $Aq_{1+2}b_1$ is $A.Aq_{1+1}b_1$ so since is a lineorly combination of the selected vectors $Aq_{1+2}b_1$ is ~~a~~ the same linear combinations are of A times the selected vectors.

This completes the proof.

* Conclusion:

It has been shown that the standard phase variable canonical form for single-input system can be extended to multi-input systems but that the extended version is not unique. Probably the most interesting form is the one described by theorem 1.

The canonical forms developed for multi-input systems have direct analogs as canonical forms for multi-output systems, the details of their development forms for multi-output systems the result given here are straight-forward

Canonical forms for multi-variable system are a useful as a straightening point for deriving certain order general result for multi-variable systems or for initiating design considerations.

* References:

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