Topic: MAT - 535

Page:

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MSc Math : Sem IV (2021-22)

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project paper : Linear Algebra

Project : Rank of linear Transformation

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Topic:

Page:

Date:

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Linear Algebra

* Rank of linear transformation -

$$L : V \to W,$$

we want to know is there a linear Transformation or not,

$$M : W \to V$$

Such that for any vector $v \in V$ we have

$$ML v = v$$

and for any vector $w \in W$, we have

$$LMw = w$$

A linear transformation is a Special kind of function from one vector space To another.

Sign:

Topic :

Page :
Date :

so before we disscuss such transfor-mation have inverse let us first disscuss inverse of arbitary functions. when we later specilize to linear transfor-mations we will also find some noice ways to ~~cra~~ creating subspaces.

Let $f : S \to T$ Be a function from a set S to a function set T Recall that S is called the domain of f, T is called The co-domain or target of f and the set $ran(f) = im(f) = f(S) = \{f(s) / s \in S\} \subset T$

is called the range or image of f the image of f is the set of elements of T To which the function f maps i.e the things in T which you can get to by Starting in S and applying f. we Can also talk about the Pre-time image of any subset $U \subset T$.

$$f^{-1}(U) = \{s \in S / f(s) \in U\} \subset S$$

The pre image of a set U is the set of all

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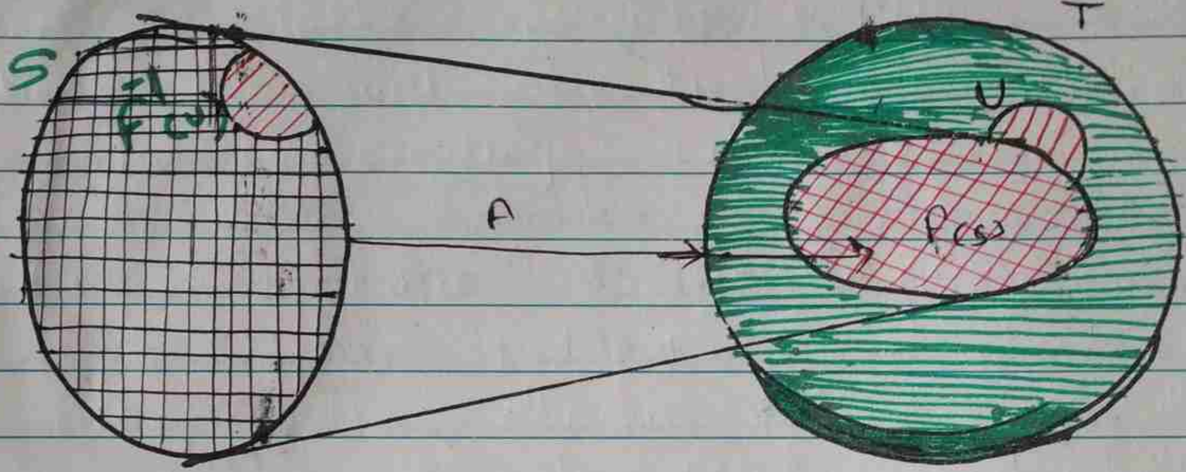
all elements of S which map to U.



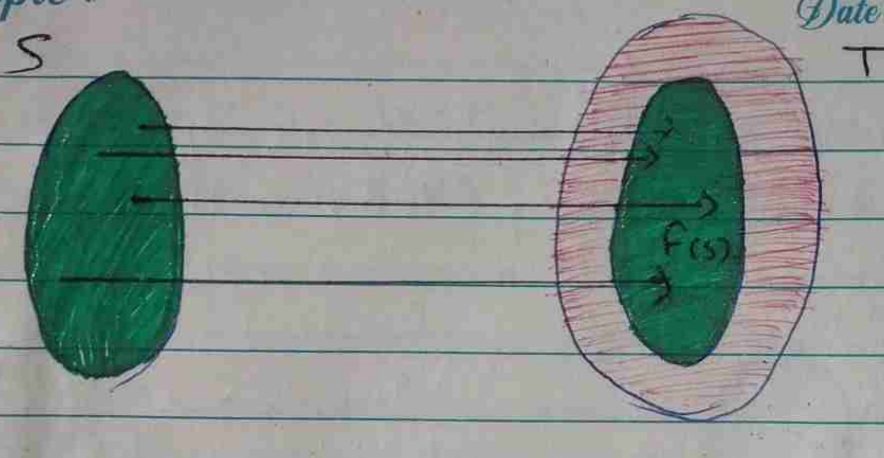
For the function $f: S \to T$ S is the domain T is the target f(s) is the image/range and $f^{-1}(U)$ is the Pre image of $U \subset T$.

The function f is one to one if different elements in S always map to diffrent elements in T that is F is one-to-one if for any elements $x \neq y \in S$, we have that $f(x) \neq f(y)$:

Topic:

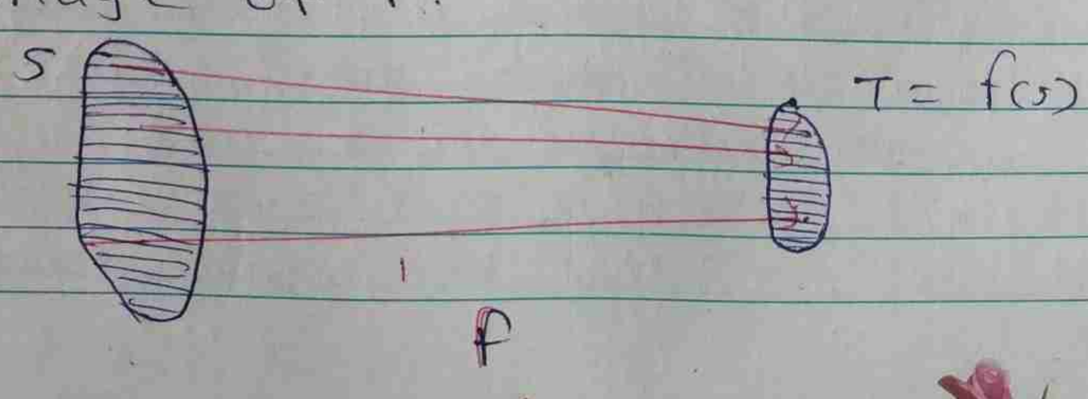
Page:

Date:



one to one function are also called injective functions. Notice that injectivity is a condition on the pre images of f

The function f is on to if every element of T is mapped to by some element of S. that is if f is onto if for any $t \in T$, There exist some $s \in S$ such that $f(s) = t$. on to functions are also called surjective functions. Notice that surjectivity is a condition on the image of f.



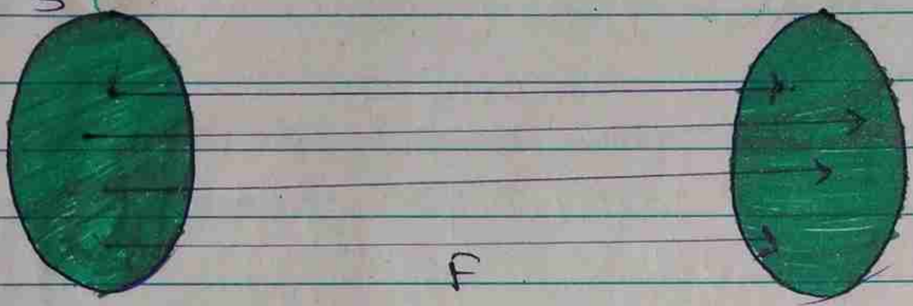
Sign:

Topic :

Page :

Date :

If f is both injective and surjective it is bijective



Theorem :-

A function $f: S \to T$ has an inverse function $g: T \to S$ If and only If it is Bijective.

Proof :

This is an if and only if statement so the Proof has two parts

a) suppose that f has an inverse function g. we need to show f is Bijective, which we Break down in to injective the function f is injective suppose that we have $s, s' \in S$ such that $f(x) = f(y)$ we must have that $g(f(s)) = s$ for any $s \in S$

Sign:

Topic :

Page :
Date :

$g(f(s)) = s$ and $g(f(s')) = s'$. But since $f(s) = f(s')$ we have $g(f(s)) = g(f(s'))$ so $s = s'$ therefore f is injective

b) The function f is surjective : Let t be any element of T we must have that

$f(g(t)) = t$. Thus $g(t)$ is an element of S which maps to t. so f is surjective

2) (Bijectivity $\Rightarrow$ Exestance of an inverse)

Suppose that f is bijective Hence f is surjective, so every element $t \in T$ has at least one pre-image Being Bijective, f is also injective so every t has no more than one pre-image. Therefore to construct an inverse function g, we ~~get~~, simply define $g(t)$ to be the unique pre-image $f^{-1}(t)$ of t.

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Linear Transformation

Let $L: V \to W$ be a linear transformation the set of all vectors v such that $Lv = 0_W$ is called the kernel of L:

$$\ker L = \{ v \in V / Lv = 0_W \} \subset V.$$