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M.Sc. Math Sem -IV

Project Paper :- Linear Algebra.

Title of project :- Studies on Canonical Forms.

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Introduction :.

In mathematics canonical, normal, or standered Form of mathematical object is a standered way of presenting that object as a mathematical expression,

often it is one which porides the simplest representation of an object and which allow it to be identified in a unique way.

In most Field a canonical form specifies a unique representation for every object. while normal form simple specifies its form, without the requirement of uniqueness.

The canonical form of positive integer in decimal ~~fraction~~ representation is a finite sequence of digits that does not begin with zero.

more generally for a class of objects on which an equivalence relation is defined a canonical form,

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Consists in the choice of a specific object in each class.

For example,

- Jordon normal form is a canonical form for matrix similarity.

- The row echelon form is a canonical form, when one considers as equivalent a matrix and left product by an invertible matrix.

- Despite this advantage canonical forms frequently depends on arbitrary choices, which introduce difficulties for testing the equality of two objects resulting on independent computation.

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Definition :

Given a set S of objects with an equivalance relation R on S a canonical form is given by designating some objects of S to be 'in canonical form' such that every object under consideration is equivalent to exactly one object in canonical form.

In other words, canonical forms in S represent the equivalence classes once and only once. To test whether two object are equivalent it then suffices to test equality on their canonical form.

Formally, a canonicalization with respect to an equivalence relation R on a set 'S' is a mapping c:

$S \to S$ such that for all $s, s_1, s_2 \in S$:

1] $c(s) = c(c(s))$ [idempotence]

2] $s_1 R s_2$ = if and only if $c(s_1) = c(s_2)$ [decisiveness].

3] $s R c(s)$ [representativeness].

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A canonical form may simply be a convention or deep theorem.
For example,
polynomials are conventionally written with the terms in descending power. it is more usual to write

$x^2 + x + 30$ than,

$x + 30 + x^2$,

although. the two forms define the same polynomial. By contrast the existence of Jordan canonical form for a matrix is a deep theorem.

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History :.

The terms canonical stems from the Ancient Greek coord kanonikos from kanon. The sense of norm standard or archetype has been used in many disciplines.

Mathematical Usage is attested in a 1738 Letter From Logan. The German term kanonische Form is attested in a 1846 paper by Eisenstein.

I now proceed to [....] the mode of reducing Algebrical function to their simplest and most symmetrical or as my admirable friend, M. Hermite well proposes to call them their canonical Forms.

In 1865 the Dictionary of science defines canonical form as, In mathematics denotes a form usually the simplest or most symmetrical to which loss of generality all function of same class can be reduced.

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Examples :

- Canonical representation of a positive integer
- Canonical form of a continued fraction.
- Linear algebra,

① Object : normal matrices over the complex number.

A is equivalent to B if :

$A = U^* BU$ for some unitary matrix U.

normal form :

Diagonal matrices.

② Object : matrices over the complex number

A is equivalent to B if :

$A = UBV^*$ for some unitary matrices U and V.

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(3) Objects: matrices over an algebraically closed field.

A is equivalent to B if.

$A = P^{-1} BP$ for some invertible matrix P.

normal Form:

Jordan normal Form.

(4) matrices over a field : object.

A is equivalent to B if

$A = P^{-1} BP$ for some invertible matrix P.

normal Form: Frobenius Form.

(5) Objects: matrices over a principle ideal domain.

A is equivalent to B if

$A = P^{-1} BQ$ for some invertible matrices P and Q.

normal form: Smith Form.

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- Algbra:.

Objects:: Finitely generated R-modules with R an principal ideal domain.

A is equivalent to B if.
A and B are isomorphic as R-modules

Normal Form:
primary decomposition or Invariant factor decomposition.

- Geometry:
In analytic geometry:
- The equation of line: $Ax + By = C$ with $A^2 + B^2 = 1$ and $C \geq 0$.
- The equation of a circle:
$(x-h)^2 + (y-k)^2 = r^2$.

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- **Integrable System :-**
  Every differential manifold has a contangent bundle. that bundle can always be endowed with a certain differential form called the canonical one-form.

- **Dynamical system :-**
  The study of dynamical system overlaps with that of integrable system. there one has the idea of normal form.

- **Three dimensional geometry :-**
  In the study of manifolds in three dimensions one has the First Fundamental form, the second Fundamental form and third Fundamendl form.

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