

Date: 21/06/2022

Page: \_\_\_\_\_

Topic: \_\_\_\_\_

Student Name: Vyankat Shankar Udase

College Name: Shrikrishna Mahavidyalaya,  
Gunjoti

Class: M. Sc. Second Year

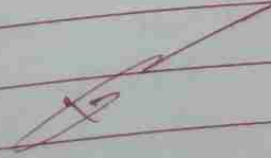
Project Name: Operation Research I I

Title of the Project: Quadratic Programming

PRN Number: 2015015200600311

Seat Number: CT-0401582

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Department of Mathematics,  
Shrikrishna Mahavidyalaya, G. Y. K.  
Tq. Omarga Dist. Osmanabad  
(M. S. J. 413006)

Date: \_\_\_\_\_

Page: \_\_\_\_\_

01

Topic: Quadratic programming

Name: Vyankat Shankar Udase

Project name: Operation Research - II

Title of the project: Quadratic programming

Quadratic programming is the process of solving certain mathematical optimization problems involving quadratic functions. Specifically, one seeks to optimize a multivariate quadratic function subject to linear constraints on the variables. Quadratic programming is a type of nonlinear programming.

"Programming" in this context refers to a formal procedure for solving mathematical problems. This usage dates on the 1940s and is not specifically tied to the more recent notion of "computer programming". To avoid confusion, some practitioners prefer the term "optimization" e.g. "quadratic optimization".



Date: \_\_\_\_\_ Page: 02

Topic: Quadratic Programming

### Problem formulation.

The quadratic programming problems with  $n$  variables and  $m$  constraints can be formulated as follow. Given.

- A real valued,  $n$ -dimensional vector  $C$ ,
- An  $n \times n$  dimensional real symmetric matrix  $Q$ ,
- An  $m \times n$ -dimensional real matrix  $A$  and
- An  $m$ -dimensional real vector  $b$ ,

The objective of Quadratic Programming is to find an  $n$ -dimensional vector  $x$ , that will

$$\text{minimize } \frac{1}{2} x^T Q x + C^T x$$

$$\text{subject to } Ax \leq b,$$

Date: \_\_\_\_\_

Page: 03

Topic: Quadratic Programming

where  $x^T$  denotes the vector transpose of  $x$ , and the notation  $Ax \leq b$  means that every entry of vector  $Ax$  is less than or equal to the corresponding entry of vector  $b$  (component-wise inequality).

### Least Squares

As a special case when  $Q$  is symmetric positive-definite, the cost function reduces to least squares:

$$\text{minimize } \frac{1}{2} \|Ax - d\|^2$$

$$\text{subject to } Ax \leq b,$$

where  $Q = R^T R$  follows from the

Cholesky decomposition of  $Q$  and  $c = -R^T d$ .

Conversely, and such constrained least squares program can be equivalently framed as a QP, even for generic non-square  $R$  matrix.



Date : \_\_\_\_\_

Page : 04

Topic : Quadratic Programming

When minimizing a function  $f$  in the neighborhood of some reference point  $x_0$ ,  $Q$  is set to its Hessian matrix  $H(f(x_0))$  and  $c$  is set to its gradient  $\nabla f(x_0)$ . A related programming problem, quadratic programming, can be posed by adding quadratic constraints on the variables.

### Solution methods

For general problems a variety of methods are commonly used, including

- interior point,
- active set,
- augmented Lagrangian,
- conjugate gradient,
- gradient projection,
- extensions of the simplex algorithm.

in the case in which  $Q$  is positive definite, the problem is a special

## Quadratic programming

Cases of the more general field of convex optimization.

### Equality Constraints

Quadratic programming is particularly simple when  $Q$  is positive definite and there are only equality constraints; specifically the solution process is linear. By using Lagrange multipliers and seeking the extremum of the multipliers and seeking, Lagrangian, it may be readily shown that the solution to the equality constrained problem.

$$\text{minimize } \frac{1}{2} x^T Q x + c^T x$$

$$\text{subject to } E x = d$$

is given by the linear system

$$\begin{bmatrix} Q & E^T \\ E & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} -c \\ d \end{bmatrix}$$