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Department of Mathematics Msc part - II
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Project paper :- Mechanics

paper No :- MAT-512

project topic :- Studies on two dimensional motion
of rigid bodies.

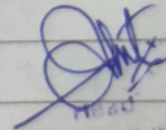
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Introduction:-

Two dimensional (2D) motion mean motion that takes place in two different directions (or coordinates) at the same time.

The simplest motion would be an object moving linearly in one dimension. An example of linear movement would be a car moving along a straight road or a ball thrown straight up from the ground.

examples of rigid body \Rightarrow smoke, fire, water, wind, leaves, cloth, magnets, flocks, fish, insects, crowds etc.

examples of two dimensional :- ① motion in a plane
② An Ant moving on top surface of desk.

2D motion: when the object travels in x and y coordinates with a constant velocity.

* study on 2-dimensional motion of rigid body.

Rotational motion of a rigid body :-

Rotational motion is more complicated than linear motion & only the motion of rigid bodies will be considered here.

A rigid body is an object with mass that holds a rigid shape, such as a phonograph turntable, in contrast to the sun which is a ball of gas. Many of the eqⁿ for the mechanics

* Angular velocity & angular acceleration

The angular displacement of a rotating wheel is the angle betⁿ the radius at the beginning & the end of the given time interval. The SI unit are radians. The avg. angular velocity (ω , Greek letter omega), measured in radians per second,

radians per second, is

$$\omega = \frac{\text{angular displacement}}{\text{elapsed time}} = \frac{\theta}{t}$$

The angular acceleration (α , Greek letter alpha) has the same form as the linear quantity.

$$\alpha = \frac{\text{change in angular velocity}}{\text{elapsed time}} = \frac{\omega_f - \omega_i}{t}$$

α is measured in radians/second/second or rad/sec^2 .

Kinematics eqⁿ for rotational motion at constant angular acceleration are.

Consider a wheel rolling without slipping in a straight line. The forward displacement of the wheel is equal to the linear displacement of the point fixed on the rim. As can be shown in figure, $d = s = r\theta$.

A wheel rolling without slipping. In this case the forward speed of the wheel is $v = d/t = (r\theta)/t = r\omega$ where r is the distance from the center of rotation to the point of the velocity is tangent to the path of the

point of rotation. The avg. forward acceleration of the wheel is $a_T = r(\omega_f - \omega_0) / t = r\alpha$. This component of the acceleration is tangential to the pt of rotation & represents the changing speed of the pt object. the direction is the same as the velocity vector.

The radial component of the linear acceleration is $a_r = v^2 / r = \omega^2 r$.

$$r = v^2 / \omega^2 = \omega^2 r$$

Torque :-

It is easier to open a door by pushing on the edge farthest from the hinges than by pushing in the middle. It is intuitive that the magnitude of the force applied & the distance from the point of application to the hinges affect the tendency of the door to rotate. This physical quantity, Torque $\tau = r F \sin \theta$.

where F is force applied, r is distance from the pt of application to the centre of the rotation.

θ is the angle from r to F .

Moment of Inertia :-

Newton's second law into the definition for torque with θ of 90° & use the relationship betⁿ linear acceleration & tangential angular acceleration to obtain $\tau = r F = r m a = m r^2 (a/r) = m r^2 \alpha$.

The quantity mr^2 is defined as moment of inertia of a point mass about the centre of rotation. Imagine two objects of the mass. The first object might be a heavy ring supported by struts on an axle like a flywheel. The second object could have its mass close to the centre axis. Even though the masses of two objects are equal, it is intuitive that the flywheel will be more difficult to push to a high number of revolutions per second because not only the amount of mass affects the ease in initiating rotations for a rigid body. The general definition of moment of inertia, is also called, **Rotational inertia**, rigid body. $I = \sum m_i r^2$

$$\text{SI unit} = \text{kg} \cdot \text{m}^2.$$

→ **Angular momentum:-**

rotation momentum that is conserved in the same way that linear momentum is conserved. For a rigid body, the angular momentum (L) is the product of the moment of inertia & angular velocity;

$L = I\omega$. For a pt of mass, angular momentum can be expressed as the radius (r):

$$L = mvr, \quad \text{SI unit} = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}.$$

Rotational kinetic energy, work & power.

kinetic energy, work & power are defined in rotational terms as $K.E = (1/2)I\omega^2$

$W = \tau\theta, P = \tau\omega$

Comparison of dynamics eqⁿ for linear & rotational motion:-

The dynamic relations are given to compare the eqⁿ for linear & rotational motion

	linear motion	Rotational motion
Newton's 2 nd law	$F = ma$	$\tau = I\alpha$
Momentum	$p = mv$	$L = I\omega$
Work	$W = F\Delta x$ or $W = \int F \cdot dx$	$W = E\Delta\theta$ or $W = \int \tau d\theta$
kinetic energy	$K.E = \frac{1}{2}mv^2$	$K.E = \frac{1}{2}I\omega^2$
Power	$P = Fv$	$P = \tau\omega$

Reference:- google & mechanics book, & Ed4Rev app.