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Class :- M.Sc. SY

Year :- 2021-22

Semester :- IV

Paper :- Mechanics (MAT-512)

Project Title :- The fixed end point Problem

Subject Teacher

13/20

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The fixed end Point Problem for n unknown function

Let $F(x, y_1, z_2, \dots y_n\ z_1, z_2, \dots z_n)$ be a function with continious first and Second derivative with respect to all its arguments consider the Problem of finding necessary Condition for Extrimum of function of the form

$$\text{If } J[y_1, y_2 \dots y_n] \doteq$$

$$\int_a^b F(x, y_1, z_2 \dots y_n, y_1', y_2' \dots y_n')\, dx$$

which depends on n indedepedents contineously differentiable function $y(x), y(x) \dots y_n(x)$

Satisfying the Boundry Condition

$$y(a) = A_i \cdot y(b) = B_i \quad (i = 1, 2, \dots n)$$

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Fixed end Point Problems

we shall start with the simplest function maximization Problem and show how to solve by finding the First veriation and deriving The Euler- Lagrange Equation

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial f}{\partial y} = 0$$

Formulation -

Define the formulation $F : C^2[x_0, x_1] \Rightarrow$

$$F\{y\} = \int_{x_0}^{x_1} f(x, y, y')\, dx$$

where f is assumed to be function with (atleast) contineous second order Partial derivatives with respect to x, y and y'.

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Simplification :-

Integrate the second term by parts.

$$\delta F(\eta, y) = \int_{x_0}^{x_1} \left[\eta \frac{\partial f}{\partial y} + \eta' \frac{\partial f}{\partial y'}\right] dx$$

$$= \left[\eta \frac{\partial f}{\partial y'}\right]_{x_0}^{x_1} + \int_{x_0}^{x_1} \eta \left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)\right] dx$$

But note that by Problem defination $\eta \in H$, and so $\eta(x_0) = \eta(x_1) = 0$ and so that first term is zero. The function inside the integral exists, and is continuous by our assumption that f has two conditinuous derivatives so for

$$E(x) = \left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)\right]$$

$$\delta F(\eta, y) = \int_{x_0}^{x_1} \eta(x) E(x) dx = (\eta E)^2 = 0$$

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Euler Lagrange Equation

Theorem :-

let $F : C^2[x_0, x_1] \to \mathbb{R}$ be a functional of the form $F\{y\} = \int_{x_0}^{x_1} f(x, y, y')\,dx$

where f has Contineous partial derivatives of second order with respect to x, y, and y', and $x_0 < x_1$ let

$$S = \{y \in C^2[x_0, x_1] \mid y(x_0) = y_0 \text{ and } y(x_1) = y_1\},$$

where y_0 and y_1 are real numbers

If $y \in S$ is and ~~eteme~~ extremal For F, then for all $x \in [x_0, x_1]$

$$\boxed{\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y} = 0} \Leftarrow \text{The Eluur Lagrange Eq}^n$$

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Proof of Euler-Lagrange Equation:-

As noted earlier at an extremal the first variation

$$\delta F(\eta, y) = \langle \eta(x), E(x) \rangle^2$$

$$= \int_{x_0}^{x_1} \eta(x) E(x) dx = 0$$

For all $\eta(x) \in H$. From Lemma we can state that,

$$E(x) = \left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)\right] = 0$$

which is the ~~p~~ Euler Lagrange Equation.

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Special case.

when f depends only on y' the EL Equation Simplify to

$$\frac{\partial f}{\partial y'} = \text{constant.}$$

An Example of this is calculating geodesics in the plane (which we all know ~~the~~ are straight lines)

f depends only on y'

Geodesics in the Plane are a special case of $f = f(y')$ with no Explicit dependance on y Apply the chain Rule to E-L equation and we get

$$\frac{d}{dx} \frac{\partial f}{\partial y'} = 0$$

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$$\frac{d^2 f(y')}{d^2 y'^2} \cdot \frac{dy'}{dx} = 0$$

$$\frac{d^2 f(y')}{d^2 y'^2} \cdot y'' = 0$$

so one of ~~two~~ the two following must be true

$$f''(y') = 0$$

$$y'' = 0.$$

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