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Introduction.

In mathematics especially algebric geometry the decomposition theorem is a set. results concerning the cohomology of algebric varieties.

$f : \to X \to Y$. of relative dimension d between two projective varieties

$$- \cup \eta^i : R^{d-i} f_* (Q) \xrightarrow{\cong} R^{d+i} f_* (Q)$$

Here η is the fundamental class of a hyperplane section, f_* is the direct image (pushforward) & $R^n f_*$ is the n-th derived functor of the direct image. This derived functor measures the n-th cohomologies of $f^{-1}(U)$, for $U \subset Y$. In fact, the particular case when Y is a point, amounts to the isomorphism:

$$- \cup \eta^i : H^{d-i} (X, Q) \xrightarrow{\cong} H^{d+i} (X, Q)$$

This hard Lefschetz isomorphism induces canonical isomorphisms.

$$Rf_*(Q) \xrightarrow{\simeq} \bigoplus_{i=-d}^{d} R^{d+i} f_*(Q)[-d-i]$$

first proved by Beilinson, & Deligne. Their proof is based on the usage of weights on l-adic sheaves is positive characteristic.

A different proof using mixed Hodge modules was given by Saito. A more geometric proof; based on the notation of semismall maps was given by de Cataldo & Migliorini.

For semismall maps, the decomposition theorem also applies to Chow motives.

Application of a the theorem.

Cohomology of a Rational Lefschetz pencil.

- Consider a rational morphism $f: X \to P'$ ~~from~~ from a smooth quasi-projective variety give by

$[f_1(x) : f_2(x)]$. If we set the vanishing locus of $f_1 f_2$ as Y then there is an induced morphism.

First Decomposition Theorem.

- For every $A \in F(X)$,

$$A = \bigcup_{a \in [0,1]} {}_aA,$$

where ${}_aA$ is defined by (2.1) and $\cup$ denotes the standard fuzzy union.

- Proof:- For each particular $x \in X$, let $a = A(x)$. Then,

$$\left(\bigcup_{a \in (0,1)} {}_aA \right)(x) = \sup_{a \in (0,1)} {}_aA(x)$$

$$= \max\left[\sup_{a \in [0,a]} {}_aA(x), \sup_{a \in (a,1]} {}_aA(x) \right]$$

For each $a \in (a, 1)$ we have $A(x) = \alpha < a$ and, therefore, ${}_aA(x) = 0$. On the other hand, for each $a \in [0, a]$, we have $A(x) = a \geq a$, therefore, ${}_aA(x) = a$. Hence, For each since the same argument is valid for each $x \in X$, the validity of (2.2) is established.

To illustrate the application of this theorem, let us consider a fuzzy set A with the following membership function of triangular shape. fig 2.2.

$$A(x) \begin{cases} x-1 & \text{when } x \in [1,2] \\ 3-x & \text{when } x \in (2,3) \\ 0 & \text{otherwise.} \end{cases}$$

for each $\alpha \in (0,1)$, the α-cut of A is in this case the closed interval.

Examples q sets αA and α^{A} for thee value q α are shown in fig 2.2 According to theorem 2.5, A is obtained 2.5, A is obtained by taking the standard fuzzy union q sets

αA for all $\alpha \in [0, 1]$.

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