

Shei Keishna Mahavidyalaya  
Gunjoti

Department of Mathematics.

M.Sc Mathematics sem - IV  
Project.

\* Student name - Bhakti Keushanat  
Kolekar.

\* Std:- Msc Mathematics sem - IV

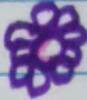
\* Subject:- Fuzzy Mathematics; IV  
MAT - 534.

\* Project Name - Studies on Decomposi-  
tion Theorems.

\* PRN No:- 2027025200850477.

\* Seat No:- CTD401568



2) Statement.  4

3) proofs 8

4) Application of the theorem. 9

5) References.

6) Diest decomposition theorem. 10

7) References. 13.

## Introduction.

In mathematics especially algebraic geometry the decomposition Theorem is a set of results concerning the cohomology of algebraic varieties.

$f: X \rightarrow Y$  of relative dimension  $d$  between two projective varieties

$$- \cup \eta^i: H^{d-i}(X, \mathbb{Q}) \xrightarrow{\cong} H^{d+i}(Y, \mathbb{Q})$$

Here  $\eta$  is the fundamental class of a hyperplane section,  $f_*$  is the direct image (pushforward) &  $R^n f_*$  is the  $n$ -th derived functor of the direct image. This derived functor measures the  $n$ -th cohomologies of  $f^{-1}(U)$ , for  $U \subset Y$ . In fact, the particular case when  $Y$  is a point, amounts to the  $\cong$  isomorphism.

$$- \cup \eta^i: H^{d-i}(X, \mathbb{Q}) \xrightarrow{\cong} H^{d+i}(X, \mathbb{Q})$$

This hard Lefschetz isomorphism induces canonical isomorphisms.

$$Rf_* (\mathbb{Q}) \xrightarrow{\cong} \bigoplus_{i=-d}^d R^{d+i} f_* (\mathbb{Q}) [-d-i]$$

first proved by Beilinson, & Deligne.  
Their proof is based on the usage of  
weights on  $I$ -adic sheaves in  
positive characteristic.

A different proof using  
mixed Hodge modules was given by  
Saito. A more geometric proof, based  
on the notation of semismall maps  
was given by de Cataldo & Migliorini.

For semismall maps, the decomposition  
theorem also applies to Chow  
motives.

Application of the theorem.

Cohomology of a Rational Lefschetz pencil.

- Consider a rational morphism  $f: X \rightarrow \mathbb{P}^1$  from a smooth quasi-projective variety given by

$[f_1(x): f_2(x)]$ . If we set the vanishing locus of  $f_1 f_2$  as  $Y$  then there is an induced morphism.

## First Decomposition Theorem.

- For every  $A \in F(X)$ ,

$$A = \bigcup_{\alpha \in [0,1]} \alpha A,$$

where  $\alpha A$  is defined by (2.1) and  $\bigcup$  denotes the standard fuzzy union.

- proof: - For each particular  $x \in X$ , let  $a = A(x)$ . Then,

$$\left( \bigcup_{\alpha \in [0,1]} \alpha A \right) (x) = \sup_{\alpha \in [0,1]} \alpha A(x)$$

$$= \max \left\{ \sup_{\alpha \in [0,1]} \alpha A(x), \sup_{\alpha \in [0,1]} \alpha A(x) \right\}$$

For each  $\alpha \in (a, 1)$  we have  $A(x) = a < \alpha$  and, therefore,  $\alpha A(x) = 0$ . On the other hand, for each  $\alpha \in [0, a]$ , we have  $A(x) = a \geq \alpha$ , therefore,  $\alpha A(x) = \alpha$ . Hence,



For each since the same argument is valid for each  $x \in X$ , the validity of (2.2) is established.

To illustrate the application of this theorem, let us consider a fuzzy set  $A$  with the following membership function of triangular shape. fig 2.2.

$$A(x) = \begin{cases} x-1 & \text{when } x \in (1, 2] \\ 3-x & \text{when } x \in (2, 3) \\ 0 & \text{otherwise.} \end{cases}$$

For each  $\alpha \in (0, 1)$ , the  $\alpha$ -cut of  $A$  is in this case the closed interval.

Examples of sets  $\alpha A$  and  $\alpha A$  for three values of  $\alpha$  are shown in fig 2.2. According to Theorem 2.5,  $A$  is obtained by taking the standard fuzzy union of sets

$\alpha A$  for all  $\alpha \in [0, 1]$ .

## \* References.

1) ^ Deligne, Pierre (1968) "Théorème de  
Stiefel-Whitney. Cotes de dégené-  
scence de suites spectrales" Publ Math,  
Inst. Hautes Etudes  
Sci 35: 107-126 + doi: 10.1007/BF02698928  
S2CID 121086385, Zbl 0154.22501

2) ^ Beilinson, Alexander A.; Bernstein,  
Joseph; Deligne, Pierre (1982). "Faisceaux  
algébriques", Astérisque (in French).  
Société mathématique de France, Paris.  
100.

B) Fuzzy Sets AND Fuzzy Logic.  
Theory and Applications. George J.  
Klir / Bo Yuan.  
Prentice Hall PTR Upper Saddle River,  
New Jersey 07458.

includes bibliographical references and  
index ISBN 0-13-101171-5

2. Fuzzy sets 2 Fuzzy logic J. Yuan, Bo.  
II Title QA248 . K487 1995  
5 113 -dc20. / 9430398  
CIP.