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FUZZY MATHEMATICS

* Studies On Decomposition Theorems *

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FUZZY MATHEMATICS

* Studies on Decomposition Theorms *

In mathematics, especially algebraic geometry the decomposition theorem is a set of results concerning the cohomology of algebraic varieties.

Statement -
Decomposition for smooth proper maps.

The first case of the decomposition theorem arises via the hard lefschetz theorem which gives isomorphisms, for a smooth proper map $f:x \to y$ of relative dimension d between two projective varieties.

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$- \cup_{\eta}^{i} : R^{d-i} f_{*}(\mathbb{Q}) \xrightarrow{\cong} R^{d+i} f_{*}(\mathbb{Q})$

Here η is the fundamental class of a hyperplane section. f_* is the direct image (pushforward) and $R^n f_*$ is the n-th derived functor of the direct image. this derived functor measures the n-th cohomologies of $f^{-1}(U)$, for $U \subset Y$ - In fact, the particular case when Y is a point, amounts to the isomorphism.

$- \cup_{\eta}^{i} : H^{d-i}(X, \mathbb{Q}) \xrightarrow{\cong} H^{d+i}(X, \mathbb{Q})$

This hard Lefschetz isomorphism induces cononical isomorphisms.

$$Rf_*(\mathbb{Q}) \xrightarrow{\cong} \bigoplus_{i=-d}^{d} R^{d-i} f_*(\mathbb{Q})[-d-i].$$

more over. the she aves $R^{d+i} f_* \mathbb{Q}$ appearing in this decomposition are local systems. locally free sheaves of $\mathbb{Q}$-vector spaces, which are moreover semisimple i.e., a direct sum of local systems without nontrivial local subsystems

Decomposition For Proper Maps

The decomposition theorem generalizes this fact to the case of a proper but not necessarily smooth map $f: X \to Y$ between varieties. In a nutshell, the results above remain true when the motion of local systems is replaced by perverse sheaves.

The hard Lefschetz theorem above takes the following form there is an isomorphism in the derived category of sheaves on Y:

$$PH^{-i}(Rf_* \mathbb{Q}) \cong PH^{+i}(Rf_* \mathbb{Q}).$$

where Rf_* is the total derived functor of f_* and PH^i is the i-th truncation with respect to the perverse t- structure.

moreover, there is an isomorphism

$$Rf_* IC_X^\cdot \cong \bigoplus_i {}^pH^{-i}(Rf_* IC^\cdot X)[-i].$$

where the summands are semi-simple pervers-sheaves, meaning they are direct sums of push-forwards of intersection cohomology sheaves.

If x is not smooth, then the above results remain true when $\mathbb{Q}[\dim X]$ is replaced by the intersection cohomology complex IC.

Proofs -

The decomposition theorem was first proved by Beilinson, Bernstein and Deligne their proof is based on the usage of weights on ladic sheaves is positive characteric proof based on the notion of semismall maps was given by de cataldo and migliorini.

For semismall maps, the decomposition theorem also applies to chowmotives.

Applications of the theorem -

cohomology of a Rational Lefschetz pencil →

consider a rational morphism $f : x : \to P^1$ from a smooth quasi-projective variety given by $[f_1(x) : f_2(x)]$. If we set the vanishing locus of $f_1 \cdot f_2$ as y then there is an induced morphism $\check{x} = Bl_y(x) \to P^1$. we cam compute the cohomology of X from the intersection cohomology of $Bl_y(X)$ and subtracting off the cohomology from the blowup along Y. This can be done using the perverse spectral sequence

$$E_2^{l,m} = H^l(P^1; {}^pH^m(IC_{\dot{X}}(\mathbb{Q})) \rightarrow IH^l.$$

References -

① Deligne, Pierre (1968), "Theoreme de lefschetz et criteres de degenerescence de suites spectrales."