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Class :- M.Sc. SY

Year :- 2021-22

Semester :- IV

Paper :- Fuzzy mathematics (MAT -534)

Project Title :- studies on fuzzy sets

Subject Teacher

$\frac{15}{20}$

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Project Paper -
Fuzzy Mathematics

MAHAGURU Page No.
Date 21.06.22

MAT - 534

Studies on Fuzzy Sets Versus Crisps sets

- Introduction -

The capability of fuzzy sets to express gradual transitions from membership to non-membership and vice versa. has been broad utility.

In fact, the basic concept of the fuzzy sets, a concept that is both simple and intuitively pleasing and that forms, in essence, a generalization of the classical or crisp set
The Crisp is defined in such a way as to dichotomize the individuals in som- given universe of discourse into two group.
members (those that certainly belong in the set)

non-members (those that certainly do not)
A sharp, unambiguous distinction exists a betn members and non-members

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A fuzzy set can be defined mathematicall by assigning to each possible individua in the universe of discourse a value representing its grade of membership i the fuzzy set.

Research on the theory of the fuzzy set has been growing steadily since the incep of the theory in the mid-1960s

The body of concepts and results perta to the theory is now quite impressive. Research on a broad variety of applicat has also been very active and has produ results that are perhaps even more impressive.

* Crisp sets - An Overview

The aim of which this text to introduce main components of fuzzy set theory and Overview some of its application. To the distinguish betn fuzzy sets and classical (nonfuzzy) sets, we refer to the lat as crisp sets!

This name is now generally accepted the literature.

In our presentation, we assume that the is familar with fundamental of the theory of crisp sets

We include this section into the text solely to refresh the basic concepts of crisp sets and to introduce notation and the terminology useful in our dissussion of fuzzy sets.

The following general symbols are the employed, as needed, throughout the text.

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (the set of all integers),

$\mathbb{N} = \{1, 2, 3 \ldots\}$ (the set of all positive integers or natural numbers},

$\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ (the set of all non-negative integers).

$\mathbb{N}_n = \{1, 2, \ldots n\}$.

$\mathbb{N}_{0n} = \{0, 1, \ldots n\}$,

$\mathbb{R}$: the set of all real numbers.

$\mathbb{R}^+$: the set of all non-negative real number [a,b], (a,b], [a,b), (a,b) : closed, left-, right-open, open interval of real numbers between a and b respectively.

$(x_1, x_2, \ldots, x_n)$: ordered n-tuple of a elements $x_1, x_2, \ldots, x_n$.

In addition, we use "iff" as a shorthand expression of "if and only if", and the standard symbols, $\exists$ and $\forall$ are used for the existential quantifier and universal quantifier respectively.

Sets are denoted in this text by upper-case letters and their members by lower-case letters.
The letter X denotes the universe of a discourse, or universal set.
This set contains all the possible elements of concern in each particular context or application from which sets can be formed.

To indicate that an individual object x is a member or element of a set A, we write

$$x \in A$$

whenever x is not an element of a set A we write

$$x \notin A$$

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There are three basic methods by which sets can be defined within a given universal set X.

1 A set is defined by naming all its members (the list method)
this method can be used only for finite sets A, whose members are $a_1, a_2 \cdot\cdot a_n$ is usually written as

2 A set is defined by a property satisfied by its members (the rule method). A common notation expressing this method is

$$A = \{x \mid P(x)\}.$$

where the symbol | denotes the phrase "such that" and P(x) designates a proposition of the form, x has the property P"
That is, A is defined by this notation as the set of all elements of X for which the proposition P(x) is true. It is required that the property P be such that for any given $x \in X$, the proposition P(x) is either true or false.

3. A set is defined by a function, usually called a characterstics function, that declares which elements of X are members of the set and which are not

That is the characterstics function maps elem
of X to elements of the set $\{0,1\}$, which
formally expressed by

$$\chi_A : X \to \{0,1\}.$$

for each $x \in X$ when $\chi_A(x) = 1$, x is de
to be member of A : when $\chi_A(x) = 0$, x
declared as a nonmember of A.

A set whose elements are themselves s
is often reffered to as a family set. It c
be defined in the form

$$\{A_i \mid i \in I\},$$

where i and I are called the set index a
the index set, respectively. Because the in
is used to reference the set A_i, the fami
of sets is also called an indexed set. In th
text, families of sets are usually denoted b
Script Capatical letter

$$\mathcal{A} = \{A_1, A_2 \cdots A_n\}$$

If every member of set A is also a member of set B (ie, if $x \in A$ implies $x \in B$), then A is called a subset of B, and this is written as

$$A \subseteq B.$$

Every set is a subset of itself, and every set is a subset of the universal set.
If $A \subseteq B$ and $B \subseteq A$ then A and B contain the same members.
They are then called equal sets this is denoted by

$$A = B$$

To indicate that A and B are not equal we write

$$A \neq B.$$

If both $A \subseteq B$ and $A \neq B$; then B contains at least one individual that is not a member of A.
In this case, A is called a proper subset of B, which is denoted by

$$A \subset B$$

when $A \subseteq B$, we also say that A is included in B.
The family of all subsets of a given set A is called the power of set A, and it is usually denoted by $P(A)$.

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The family of all subsets of $P(A)$ is call[ed]
a second order power set of A;

it is denoted by $P^2(A)$ The family of a[ll]
Subsets of $P(A)$ is called a Second ord[er]
power set of A,

it is denoted by $P^2(A)$, which strands f[or]
$P(P(A))$. Similarly, higher order powers
$P^3(A)$, $P^4(A)$ ... Can be defined

The number of members of a Finite set A [is]
called the Cardinality of A and is denote[d]
by $|A|$. when A is finite, then

$$|P(A)| = 2^{|A|}, |P^2(A)| = 2^{2^{|A|}} \text{ etc}$$

The relative complement of a set A with
respect to set B is the set containing of
the members of B that are not also me[mbers]
of A. This can be written B − A. Thus

$$B-A = \{x \mid x \in B \text{ and } x \notin A\}.$$

If the set B is the universal set, the comp[le]ment is absolute and is usually denoted by
The absolute complement is always involut[ive]
that is, taking the Complement of a compl[ement]
yields the original set or

$$\overline{\overline{A}} = A$$

The absolute complement of the empty set equals the universal set, and the absolute complement of the universal set equals the empty set that is

$$\overline{\emptyset} = X$$

and

$$\overline{X} = \emptyset.$$

The union of the set containing of all the element that belong either to set A alone, to set B alone, or to both set A and set B. this is denoted by $A \cup B$. Thus

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

The union operation can be generalized for any number of sets. For a family of sets $\{A_i \mid i \in I\}$. this is defined as

$$\bigcup_{i \in I} A_i = \{x \mid x \in A_i \text{ for some } i \in I\}$$

The intersection of sets A and B is the set containing all the elements belonging to both set A and set B.
it is denoted as

$$\bigcup_{i \in I} A_i = \{x \mid x \in A_i \text{ for some } i \in I\}$$

The intersection of sets A and B is the se
containing all the elements belonging to bo
set A and B.
it is denoted by $A \cap B$. Thus,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The generalization of intersection for a fa
of sets $\{A_i \mid i \in I\}$ is defined as

$$\bigcap_{i \in I} A_i = \{x \mid x \in A_i, \text{ for all } i \in I\}$$

The most fundamental properties of the se
operation of absolute complement, union a
intersection are summarized in table.

Where sets A, B and C are assumed to b
elements of power set $P(X)$ of a unive
set X.
Note that all the equations in this tab
that involve the set union and intersection a
arranged in pairs.
The second equation in each pair can be
obtained from the 1st by replacing ϕ, $\cup$, an
with X. '$\cap$' and '$\cup$' respectively and vice ve
We are thus concerned with pairs of
dual equation

They exemplify a general principle of a duality for each valid equation in set theory that is based on the union and intersection operations, there corresponds a dual equation also valid.
that is obtained by the above specified replacement

FUNDAMENTAL PROPERTIES OF CRISP SET OPERATIONS

Involution	$\overline{\overline{A}} = A$
Commutativity	$A \cup B = B \cup A$
	$A \cap B = B \cap A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$
	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributivity	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Idempotence	$A \cup A = A$
Absorption	$A \cup (A \cap B) = A$
	$A \cap (A \cup B) = A$
Absorption by X and $\emptyset$	$A \cup X = X$

$A \cap X = A$

Law of contradiction $A \cap \bar{A} = \emptyset$

Law of excluded middle $A \cup \bar{A} = X$

De Morgan's laws $\overline{A \cap B} = \bar{A} \cup \bar{B}$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

Elements of the power set $P(X)$ of a u rsal set X (or any subset of X) can Ordered by the set inclusion $\subseteq$ The ord which is only partial forms a lattice in w the join (least upper bound supremu and meet (gratest lower bound influm) any pair of sets $A, B \in P(X)$ is given b $A \cup B$ and $A \cap B$ respectively

This lattice is distributive (due to t distributive properties of $\cup$ and $\cap$ listed and complemented (since each set in P has its complement in $P(X)$.

it is usually called a Boolean latti or a Boolean algebra

The connection between the two formu of this lattice, $(P(X) \subseteq)$ and $\{P(X), \cup, \cap\}$, is facilitated by t following equivalence

$A \subseteq B$ iff $A \cup B = B$ (or $A \cap B = A$)

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for any $A, B \in P(X)$.

Any two sets that the have no common members are called disjoint
That is every pair of disjoint sets A and B satisfies the equation

$$A \cap B = \phi$$

A family of pairwise disjoint non-empty subset of a set A is called a partition on A if the union of these subsets yields the original set A.

We denote a partition on A by the symbol $\pi(A)$ formally

$$\pi(A) = \{A_i \mid i \in I, A_i \subseteq A\}$$

where $A_i \neq \phi$ is a partition on A iff

$$A_i \cap A_j = \phi$$

for each pair $i, j \in I$, $i \neq j$, and

$$\bigcup_{i \in I} A_i = A.$$

Members of a partition $\pi(A)$ which are subsets of A, are usually reffered to as blocks of the partition. Each member of A belongs to one and only one block of $\pi(A)$

Given two partitions $\pi_1(A)$ and $\pi_2(A)$ ... Say that $\pi_1(A)$ is a refinement of π_2... iff each block of $\pi_1(A)$ is included in some block of $\pi_2(A)$.

The refinement relation on the set of partitions of A. $\pi(A)$ which is denoted $\leq$ (i.e $\pi_1(A) \leq \pi_2(A)$ in our case) is a partial ordering

The pair $(\pi(A) <)$ is a lattice, re... to as the partition lattice of A.

Let $A = \{A_1, A_2 \cdots A_n\}$ be a family ... sets such that

$$A_i \subseteq A_{i+1} \text{ for all } i = 1, 2 \ldots n-1$$

Then A is called a nested family and sets A_1 and A_n are called the innermost set and the outermost set, respectively. The defination can easily be extended ... infinite families

The Cartesian product of two sets - say A ... B (in this order) - is the set of all o... pair such that the first element in ... pair is a member of A, and the se... element is a member of B. formally,

$$A \times B = \{(a, b) \; a \in A, b \in B\}.$$

where $A \times B$ denotes the cartesian product.
clearly, if $A \neq B$ and A, B are non-empty, then
$A \times B \neq B \times A$

The cartesian product of a family $(A_1, A_2 \cdots A_n)$ of sets is the set of all n-tuples $(a_1, a_2 \cdots a_n)$ Such that $a_i \in A_i$ $(i = 1, 2 \cdots n)$
it is written as either $A_1 \times A_2 \times \cdots \times A_n$

or $\underset{1 \leq i \leq n}{\times} A_i$ thus

$$\underset{1 \leq i \leq n}{\times} A_i = \{ (a_1, a_2 \cdots a_n) \mid a_i \in A_i \text{ for every } i = 1, 2 \cdots n) \}$$

The cartesian product $A \times A$, $A \times A \times A$ ... are denoted by $A^2, A^3 \ldots$ respectively
Subset of cartesian product are relations
They are the subject of a chp.
If such labelling is not possible, the set is called uncountable
For instance, the set $(a \mid a$ is a real number $0 \leq a \leq 1\}$ is uncountable
Every uncountable set is infinite countable sets are classified into finite and countably infinite (also called denumerable).
As important and frequently used universal set is the set of all points in the n. dimensional Euclidean vector space $\mathbb{R}^n$

for some $n \in N$ (i.e all n-tuples of a real numbers sets defined in terms of $\mathbb{R}$ are often required to posses a property referred to as convexity A set A in $\mathbb{R}$ is called convex iff, for every pair of points

$$r = \{r_i \mid i \in N_n\} \text{ and}$$
$$s = \{s_i \mid i \in N_n\}$$

in A and every real number $\lambda \in [0,$ the point

$$t = (\lambda r_i + (1-\lambda) s_i \mid i \in N_n\}$$

* FUZZY SETS : BASIC TYPES

As defined in the previous section, the characterstics function of a crisp set as a value of either 1 or 0 to each individual in the universal set thereby discriminating betn members and non-ber of the crisp set under consideration

This function can be generalized such that the values assigned to the element the universal set fall within a specified and indicate the membership grade of these element in the set in question Larger values denote higher degree of membership.

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Such a function is called a membership function and the set defined by it a fuzzy set

The most commonly used range of values of membership function is the unit interval [0, 1]. In this case, each membership function maps elements of a given universal set X, which is always a crisp set, into real numbe in [0, 1]

Two distinct notations are most commonly employed in the literature to denote a membership function. In one of them, the membership function of a fuzzy set A is denoted by μ_A that is

$$\mu_A : X \rightarrow [0, 1].$$

In the other one, the function is denoted by A and has of course, the same form

$$A : X \rightarrow [0, 1]$$

According to the 1st notation, the symbol (laber identifier name) of the fuzzy set (A) is distinguished from the symbol of its memem ship function (μ_A).
According to the second notation this distin ction is not made, but no ambiguity result